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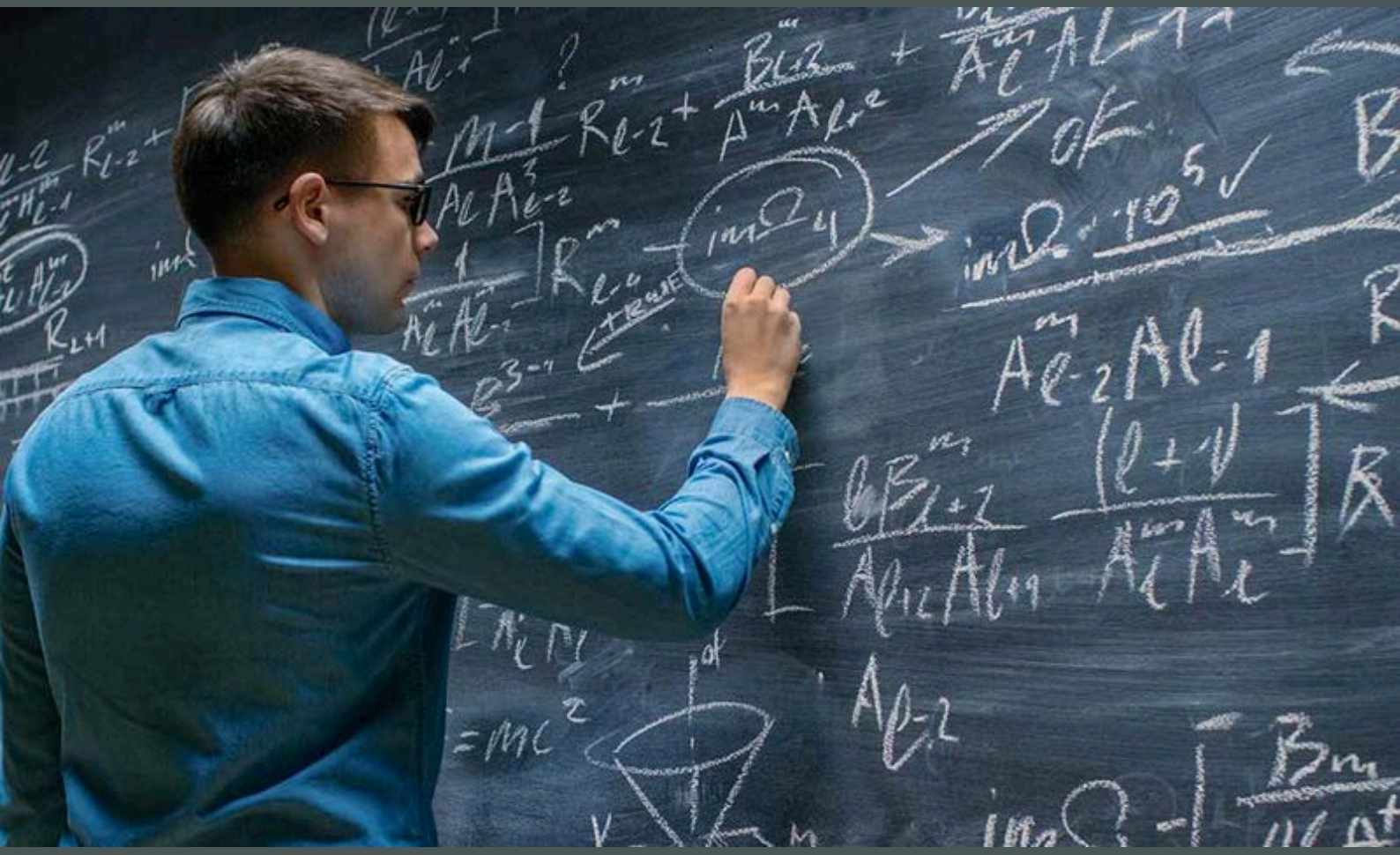

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PROBABILITY AND DISTRIBUTION

Probability may be defined as the study of random experiments. In any random experiment, there is always an uncertainty that a particular event will occur or not. As a measure of probability of occurrence of an event, a number between 0 to 1 is assigned. If it is sure that an event will occur, then we can say that its probability is 100 % or 1. If it is not sure that an event will occur, then we can say that its probability is 0% or 0. If it is not sure whether the event will occur or not, then its probability is between 0 and 1.

Let us consider an example. The probability of occurrence of 28th February in a year is 1 since it is certain to occur every year. On the other hand, the probability of occurrence of 30th February in a year is 0 since it never comes. Again, the probability of occurrence of 29th February in a year is neither 0 not 1. It is always between 0 and1. Actually, it is 1/4 since it occurs every leap year i.e., once in four years.

Therefore, from above discussion, we can write a mathematical expression for probability as

Probability of an event A,

$$P(A) = \frac{Number\ of\ \ possible\ \ favourable\ outcomes}{Total\ \ number\ of\ \ outcomes}$$

For example, let us consider the probability of getting an even number in tossing of a die.

In tossing of a die, an even number can occur in 3 ways out of 6 equally likely ways.

Therefore, $P(A) = \frac{3}{6} = \frac{1}{2} = 50\,\%$

PROPERTIES OF PROBABILITY

As discussed earlier, the sample space S contains all possible outcomes of an experiment. Also, an event is the subset of the sample space. Two events are said to be mutually exclusive if the occurrence of one of them precludes the occurrence of other.

For example, in tossing of a coin, events **Head** and **Tail** are mutually exclusive. The properties of probability may be listed as under:

Property 1 :

The probability of a certain event is unity i.e.,

$$P(A) = 1$$



Property 2 :

The probability of any event is always less than or equal to 1 and non-negative. Mathematically,

$$0 \le P(A) \le 1$$

Property 3 :

If A and B are two mutually exclusive events, then

$$P(A + B) = P(A) + P(B)$$

Property 4 :

If A is any event, then the probability of not happening of A is

$$P(\overline{A}) = 1 - P(A)$$

Where $\overline{A}$ represents the complement of event A.

Property 5 :

If A and B are any two events (not mutually exclusive events), then

$$P(A + B) = P(A) + P(B) - P(AB)$$

Where P(AB) is called the probability of events A and B both occurring simultaneously. Such an event is called joint event of A and B, and the probability P(AB) is called the joint probability.

Now, if events A and B are mutually exclusive, then the joint probability, P(AB) = 0.

Note:

- P(A+B) is also denoted as $P(A \cup B)$.
- P(AB) is also denoted as $P(A \cap B)$.

CONDITIONAL PROBABILITY OF PROBABILITY

The concept of conditional probability is used in conditional occurrences of the events. Let us consider an experiment which involves two event A and B. Now the probability of event B, given that event A has occurred is represented by P(B/A). Similarly, P(A/B) represents probability of event A given that event B ahs already occurred. Therefore, P(B/A) and P(A/B) are called *conditional probabilities*. These conditional probabilities may be defined in terms of their independent and joint probabilities as under:

The conditional probabilities of event B given that event A has already happened.



$$P\left(\frac{B}{A}\right) = \frac{P(AB)}{P(A)}$$

Where P(AB) is the joint probability of A and B.

Similarly, the conditional probability of event A given that event B has already happened.

$$P\left(\frac{A}{B}\right) = \frac{P(AB)}{P(B)}$$

At this point, it may be noted the joint probability has commutative property which states that

P (AB) = P (BA)

PROBABILITY OF STATISTCALLY INDEPENDENT EVENTS

If A and B are two events in an experiment, and possibility of occurrence of event B does not depend upon occurrence of event A, then these two events A and B are known as statistically independent events.

P(AB) = P(A) P(B)

Example

A box contains 3 Red, 4 white and 5 black balls. One ball is drawn at random. Find the probability that it is

(a) Red
(b) not Black
(c) Black or white

Solution

Let R, W and B be the events of drawing a red, a white and a black ball respectively.

(a) $P(R) = \frac{Ways\ of\ choosing\ a\ red\ ball}{Total\ no.\ of\ ways\ of\ choosing\ a\ ball} = \frac{3}{3+4+5} = \frac{3}{12} = \frac{1}{4}$

(b) Probability of not getting a black ball is $p(\overline{B})$.

$$P(\overline{B}) = 1 - P(B)$$

But $$P(B) = \frac{5}{3+4+5} = \frac{5}{12}$$

Therefore, $$P(\overline{B}) = 1 - \frac{5}{12} = \frac{7}{12}$$

(c) Probability of getting a black or white ball will be P(B + W).



$\therefore$ P(B + W) = P(B) + P(W) [$\because$ B and W are two mutually exclusive events]

$$P(B+W)=\frac{5}{12}+\frac{4}{12}=\frac{9}{12}=\frac{3}{4}$$

Example

A box contain 4 white and 3 black balls. Three balls are drawn from the box in succession. Find the probability that the first two balls are white and the third is black.

Solution

Let A_1 be the event that the first ball is a white ball.

$\therefore$ $$P(A_1)=\frac{4}{7}$$

Since there are four white balls in the box out of total, 7 balls.

Let A_2 be the event that the second ball is also a white ball.

$\therefore$ $$P\left(\frac{A_2}{A_1}\right)=\frac{3}{6}$$

Since, after the first drawing, there remain 3 white balls out of a total of 6 balls.

Let A_3 be the event that the third ball is black.

$\therefore$ $$P\left(\frac{A_3}{A_1 A_2}\right)=\frac{3}{5}$$

Since after the second drawing, there are 3 black balls out of a total of 5 balls.

Therefore the required probability is

$$P = P(A_1 + A_2 A_3) = P(A_1)\, P\left(\frac{A_2}{A_1}\right) P\left(\frac{A_3}{A_1 A_2}\right)$$

$$P = \frac{4}{7} x \frac{3}{6} x \frac{3}{5} = \frac{6}{35}$$ **Ans.**

Example

A box contain 5 white, 3 red and 2 black balls. Three balls are drawn in succession. Find the probability that the balls will be of different colours.



Solution

Let W, R, B be the events of drawing white, red and black balls, respectively. It may be observed that there are six possible ways of getting the balls of deferent colours as follows:

WRB, WBR, RWB, RBW, BWR, BRW

Now, the probability of first combination WRB will be

$$P\ (WRB) = \frac{5}{10}, \frac{3}{9}, \frac{2}{8}$$

Here , it may ne noted that

$\frac{5}{10}$ is the probability of getting first ball white

$\frac{3}{9}$ is the probability of getting second ball red

$\frac{2}{8}$ is the probability of getting third ball black

Similarly, probabilities for the other combinations may be calculated.

Since all the six combinations (events) are mutually exclusive, the required probability is

$$P = P\ (WRB) + WRB + RWB + RBW + BWR + BRW)$$

$$= P\ (WRB) + P(WBR) + P(RWB) + P(BWR) + P(BRW)$$

$$P = \left(\frac{5}{10} x \frac{3}{9} x \frac{2}{8}\right) + \left(\frac{5}{10} x \frac{2}{9} x \frac{3}{8}\right) + \left(\frac{3}{10} x \frac{5}{9} x \frac{2}{8}\right) + \left(\frac{3}{10} x \frac{2}{9} x \frac{5}{8}\right) + \left(\frac{2}{10} x \frac{5}{9} x \frac{3}{8}\right) + \left(\frac{2}{10} x \frac{3}{9} x \frac{5}{8}\right)$$

$$P = \frac{1}{4}$$ **Ans.**

Example (GTU NM 2020, 3 marks)

A can hit a target 4 times in 5 shots. B 3 times in 4 shots and C twice in 3 shots. They fire a volley. What is the probability that two shots hit the target?

Solution

Fire a volley means that A, B and C all try to hit the target simultaneously.

Two shots hit the target in one of the following ways:

(1) A and B hit and C fails to hit.
(2) A and C hit and B fails to hit.
(3) B and C hit and A fails to hit.



The chance of hitting by A,

P(A) = 4/5 and of not hitting by him P(A′) = 1 − 4/5=15

The chance of hitting by B,

P(B)=3/4 and of not hitting by him P(B′)=1 − 3/4=1/4

The chance of hitting by C,

P(C)=2/3 and of not hitting by him P(C′)=1 − 2/3=1/3

The probability of (1) is P(ABC′) = 4/5×3/4×1/3=1/5

The probability of (2) is P(ACB′) = 4/5×2/3×1/4=2/15

The probability of (3) is P(BCA′) = 3/4×2/3×1/5=1/10

Thus the probability that two shots hit the target is 1/5+2/15+1/10 = 13/30

Problem

What is the probability that the throw of two dice will be 7 or 11?

***Answer**: 2/9*

A FUNDAMENTAL THEOREM

*If a thing can be done in **m** ways and after it has been done in any one of these ways, a second can be done in **n** ways, then the two things can be done in **mn** ways.*

Example

Five balls are drawn from a bag containing 6 white and 4 black balls. What is the probability that 3 are white and 2 black?

Solution

There are in all 10 balls in the bag and total number of ways of drawing 5 from them = ${}^{10}C_5$

Now 3 white balls can be drawn from 6 in ${}^{6}C_3$ ways and two black can be drawn from 4 in ${}^{4}C_2$ ways and any group of 3 white balls can be associated with 2 black balls in ${}^{6}C_3 x {}^{4}C_2$ ways.

Hence $$p = \frac{{}^{6}C_3 x {}^{4}C_2}{{}^{10}C_5} = \frac{6x5x4}{3!} x \frac{4x3}{2!} x \frac{5!}{10x9x8x7x6} = \frac{10}{21}$$

Note: Combination formula is ${}^{n}C_r = \frac{n!}{r!(n-r)!}$



BAYES' THEOREM

If an event A can occur in conjunction with one of the n mutually exclusive and exhaustive events E_1, E_2,....E_n and if A has actually occurred then the probability that it was preceded by a particular event E_i (i = 1, 2, 3, ...) is given by

$$P(E_i / A) = \frac{P(E_i)P(A/E_i)}{\sum_{i=1}^{n} P(E_i)P(A/E_i)}$$

The symbol P(E/A) denotes the conditional probability for the event E when the event A has already occurred.

Similarly, P(A/E) denotes the conditional probability for the event A after E has occurred.

Example

Machines I and II produce 30% and 70% respectively of a factory's output. Machine I produces 3% defectives and machine II produces 4% defectives.

(a) If an item is defective what is the chance it was produced by machine I?
(b) If an item is not defective, what is the chance it was produced by machine I?

Solution

We use the following notations:

E_1 = item was produced by machine I

E_2 = item was produced by machine II

A = item is defective

B = item is not defective

Now $P(E_1) = 30/100 = 0.3; P(E_2) = 70/100 = 0.7$

$P(A/E_1) = 3/100 = 0.3; P(A/E_2) = 4/100 = 0.4$

$P(B/E_1) = 0.97; P(B/E_2) = 0.96$

Applying Bayes's theorem

(a) $$P(E_i / A) = \frac{P(E_1)P(A/E_1)}{P(E_1)(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{(0.3)(0.3)}{(0.3)(0.03) + (0.7)(0.4)}$$



$$= \frac{9}{37} = 0.24 \text{ approx.}$$

(b) $$P(E_i / B) = \frac{P(E_1)P(B/E_1)}{P(E_1)P(B/E_1) + P(E_2)P(B/E_2)}$$

$$= \frac{(0.3)(0.97)}{(0.3)(0.97) + (0.7)(0.96)}$$

$$= 291/963 = 0.30 \text{ approx}$$

Example

The chance that doctor A will diagnose a disease X correctly is 60%. The chance that a patient will die by his treatment after correct diagnose is 40% and the chance of death by wrong diagnose is 70%. A patient of doctor A who had disease X died. What is the chance that his was diagnosed correctly?

Solution

Notations:

E_1 = disease is diagnosed correctly

E_2 = disease is diagnosed incorrectly

A = patient dies

Then $$P(E_1) = 60/100 = 0.6; P(E_2) = 0.4$$

$$P(A/E_1) = 40/100 = 0.4$$

$$P(A/E_2) = 70/100 = 0.7$$

$\therefore$ $$P(E_1 / A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{(0.6)(0.4)}{(0.6)(0.4) + (0.4)(0.7)} = \frac{6}{13}$$

Problem

Companies B_1, B_2, B_3 produce 30%, 45% and 25% of the cars respectively. It is known that 2%, 3% and 2% of the cars produced from B_1, B_2 and B_3 are defective. (a) What is the probability that a car purchased is defective? (b) If a car purchased is found to be defective what is the probability that this car is produced by company B_3?

Answer: P(defective) = 0.0245; $P(B_3/D)$ = 10/49



RANDOM VARIABLE

If a **real variable** X is associated with the outcome of a random experiment is called a **random variable.** Suppose that our experiment E consists of tossing two fair coins, letting X denote the heads appearing then X is a random variable taking one of the values 0, 1, 2 with respective probabilities as follows:

$$P(X = 0) = P(T, T) = 1/4$$

$$P(X = 1) = P[(H,T),(T,H)] = 1/2$$

$$P(X = 2) = P(H, H) = 1/4$$

We have P(X 0) P(X 1) P(X 2) 1

Hence X, is a random variable taking values 0, 1, 2.

TYPES OF RANDOM VARIABLES

There are two types of random variables:

- Discrete random variables
- Continuous random variables

Discrete random variables

A random variable X is said to be discrete if it takes either finite or countably infinite values. Thus, a discrete random variable takes only isolated values, e.g.,

- Number of children in a family
- Number of cars sold by different companies in a year
- Number of days of rainfall in a city
- Number of misprints on a page
- Number of students present in a class of 50 students

Continuous random variables

A random variable X is said to be continuous if it takes any values in a given interval. Thus, a continuous random variable takes uncountable infinite values, e.g.,

- Height of a person in cm
- Life of an electric bulb in hours
- Temperature of a city in degree Celsius



DISCRETE PROBABILITY DISTRIBUTION

Probability distribution of a random variable is the set of its possible values together with their respective probabilities. Let X be a discrete random variable which takes the values x_1, x_2, … x_n. The probability of each possible outcome xi is pi = p(xi) = P(X = xi) for i = 1, 2, …, n. The number $p(x_i)$, i = 1,2, …. must satisfy the following conditions:

- $p(x_i) \geq 0$ *for all values of* i
- $\sum_{i=1}^{\infty} p(x_i) = 1$

The function $p(x_i)$ is called the **probability function or probability mass function or probability density function** of the random variable X.

The set of pairs $\{x, p(x_i)\}$, i = 1, 2, …, n is called the **probability distribution of the random variable w**hich can be displayed in the form of a table as shown below:

$X = x_i$	x_1	x_2	x_3	$...x_i$	$...x_n$
$p(x_i) = P(X = x_i)$	$p(x_1)$	$p(x_2)$	$p(x_3)$	$...p(x_i)$	$...p(x_n)$

DISCRETE DISTRIBUTION FUNCTION

Let X be a discrete random variable which takes the values x_1, x_2, … such that $x_1 < x_2 < \ldots$ with probabilities $p(x_1)$, $p(x_2)$ … such that $p(x_i) \geq 0$ for all values of i and $\sum_{i=1}^{x} p(x_i) = 1$

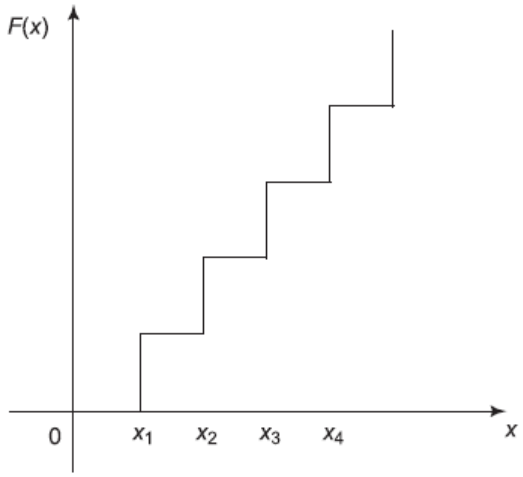
The distribution function F(x) of the discrete random variable X is defined by

$$F(x) = P(X \leq x) = \sum_{i=1}^{x} p(x_i)$$

The function F(x) is also called the **cumulative distribution function**. The set of pairs $\{x_i$, F(x)$\}$, i = 1, 2, … is called the **cumulative probability distribution**.

X	x_1	x_2	...
F(x)	$p(x_1)$	$p(x_1) + p(x_2)$	...

The graph of F(x) will be stair step form as shown below.



Example

A fair die is tossed once. If the random variable is getting an even number, find the probability distribution of X.

Solution

When a fair die is tossed,

S = {1, 2, 3, 4, 5, 6}

Let X be the random variable of getting an even number. Hence, X can take the values 0 and 1.

P(X = 0) = P(1, 3, 5) = 3/6 = 1/2

and P(X = 1) = P(2, 4, 6) = 3/6 = 1/2

Hence, the probability distribution of X is

X = x	0	1
p(X = 1)	1/2	1/2

Example

Find the probability distribution of the number of heads when three coins are tossed.

Solution

When three coins are tossed,

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

Let X be the random variable of getting heads in tossing of three coins. Hence X can take the values 0, 1, 2, 3.

P(X = 0) = P(no head) = P(TTT) = 1/8

P(X = 1) = P(one head) = P(HTT, THT, TTH) = 3/8

P(X = 2) = P(two heads) = P(HHT, THH, HTH) = 3/8

P(X = 3) = P(three heads) = P(HHH) = 1/8

Hence, the probability distribution of X is

X = x	0	1	2	3
p(X = 1)	1/8	3/8	3/8	1/8

Example (JNTUH 2019, 2 marks)

Let X denotes the number of heads in a single toss of 4 fair coins. Determine P (1 < X ≤3).

Solution

For a single toss of 4 fair coins, the sample space is as follows:

The sample space is (16 variations): HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT, THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT

P(1 < X ≤ 3) = P(X = 2) + P(X = 3) = 6/16 + 4/16 = 5/8

Example

A random variable X has the probability mass function given by

X = x	*1*	*2*	*3*	*4*
p(X = x)	*0.1*	*0.2*	*0.5*	*0.2*

Find (i) P(2 ≤x < 4) (ii) P(X is odd)

Solution

(i) P(2 ≤ x < 4) = P(X = 2) + P(X = 3) = 0.2 + 0.5 = 0.7

(ii) P(X is odd) = P(X = 1) + P(X = 3) = 0.1 + 0.5 = 0.6

Problem

A random variable X has the probability mass function given by

X = x	*1*	*2*	*3*	*4*
p(X = x)	*0.1*	*0.2*	*0.5*	*0.2*

Find (i) P(X > 2) (ii) P(X is even)

Answer: 0.7, 0.4



Example

From a lot of 10 items containing 3 defective, a sample of 4 items is drawn at random without replacement. Find the probability distribution of X, the number of defectives.

Solution

A sample of 4 can be drawn from a lot of 10 items is $^{10}C_4$ ways which are equally likely.

Let X be the number of defectives, then the number of defectives x out of 3 can be chosen in $^{3}C_x$

ways and remaining 4 – x items can be chosen from 7 items in $^{7}C_{4-x}$ ways where x = 0, 1, 2, 3, then

$$p(X = x) = \frac{^{7}C_{4-x} \cdot ^{3}C_x}{^{10}C_4}$$

Now, we can form a probability distribution of X and form a table as given below:

x	0	1	2	3
p(x)	1/6	1/2	3/10	1/30

Example

If a pair of fair dice is rolled, find the probability distribution for getting their sum 2, 3, 4, …, 12.

Solution

S = {(1,2,3,4,5,6) x(1,2,3,4,5,6)}

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6);

(2,1) (2,2) (2,3) (2,4) (2,5) (2,6);

(3,1) (3,2) (3,3) (3,4) (3,5) (3,6);

(4,1) (4,2) (4,3) (4,4) (4,5) (4,6);

(5,1) (5,2) (5,3) (5,4) (5,5) (5,6);

(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

Let X be the random variable defined as the sum of two fair dice, then

P(X = 2) = P(1,1) = 1/36

P(X = 3) = P[(l,2),(2,l)] = 2/36

P(X = 4) = P[(l. 3), (2,2) (3,1)] = 3/36



P(X = 5) = P[(l,4), (2,3), (3,2), (4,1)] = 4/36

P(X = 6) = P[(l,5), (2,4), (3,3), (4,2), (5,1)] = 5/36

P(X = 7) = P [(1,6), (2,5), (3,4) (4,3), (5,2), (6,1)] = 6/36

P(X = 8) = P[(2,6, (3,5), (4,4), (5,3), (6,2)] = 5/36

P(X = 9) = P[(3,6), (4,5), (5,4), (6,3)]= 4/36

P(X = 10) = P[(4,6),(5,5), (6,4)] = 3/36

P(X = 11) = P[(5,6), (6,5)] = 2/36

P(X = 12) = P[(6,6)] = 1/36

Hence the probability distribution of X is given as follows:

X = x	2	3	4	5	6	7	8	9	10	11	12
p(X = x)	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

Example (UTU 2013)

Five defective bulbs are accidently mixed with twenty good ones. It is not possible to just look at a bulb and tell whether or not it is defective. Find the probability distribution of the number of defective bulbs, if four bulbs are drawn at random from this lot.

Solution

Sol. Let X denote the number of defective bulbs in 4. Clearly X can take the values 0,1, 2, 3 or 4.

Number of defective bulbs = 5

Number of good bulbs = 20

Total number of bulbs = 25

P(X = 0) = P (no defective) = P (all 4 good ones)

$$= \frac{^{20}C_4}{^{25}C_4} = \frac{20x19x18x17}{25x24x23x22} = \frac{969}{2530}$$

$$\text{P (X = 2) = P(2 defective and 2 good ones)} = \frac{^{5}C_1 x\, ^{20}C_3}{^{25}C_4} = \frac{1140}{2530}$$

$$\text{P (X = 3) = P(3 defectives and 1 good one)} = \frac{^{5}C_3 x\, ^{20}C_1}{^{25}C_4} = \frac{40}{2530}$$



$$P(X = 4) = P \text{ (all 4 defectives0} = \frac{{}^{5}C_{4}}{{}^{25}C_{4}} = \frac{1}{2530}$$

The probability distribution of the random variable X is

X	0	1	2	3	4
P(X)	969/2530	1140/2530	380/2530	40/2530	1/2530

MEAN, VARIANCE AND STANDARD DEVIATION OF DISCRETE PROBABILITY DISTRIBUTION

Mean

The **mean** or average value **(μ)** of the probability distribution of a discrete random variable X is called as **expectation** and is denoted by **E(X)**.

$$\mu = E(X) = \sum_{i=1}^{\infty} x_i p(x_i) = \sum xp(x)$$

Important properties:

- E(X + k) = E(X) + k
- E(aX ± b) = aE(X) ± b
- E(X + Y) = E(X) + E(Y)
- E(XY) = E(X) E(Y)

Variance

Variance characterizes the variability in the distributions since two distributions with same mean can still have different dispersion of data about their means. Variance of the probability distribution of a discrete random variable X is given by

$$\begin{aligned} \text{Var}(X) = \sigma^2 &= E(X - \mu)^2 \\ &= E(X^2 - 2X\mu + \mu^2) \\ &= E(X)^2 - E(2X\mu) + E(\mu^2) \\ &= E(X)^2 - 2\mu E(X) + \mu^2 \\ &= E(X^2) - 2\mu\mu + \mu^2 \\ &= E(X^2) - \mu^2 \\ &= E(X^2) - [E(X)]^2 \end{aligned}$$

Important properties:



- Var (k) = 0
- Var (kX) = k^2 Var(X)
- Var (X + k) = Var (X)
- Var (aX + b) = a^2 Var(X)

Standard Deviation

Standard deviation is the positive square root of the variance.

$$SD = \sigma = \sqrt{\sum_{i=1}^{\infty} p(x_i) - \mu^2}$$

$$= \sqrt{E(X^2) - \mu^2}$$

$$= \sqrt{E(X^2) - [E(X)]^2}$$

Example (JNTUH 2018, 3 marks)

A fair coin is tossed until a head or consecutive five tails occurs. Find the discrete probability distribution. Also find the mean.

Solution

Given X denotes the number of coins. Coins is tossed until a head or five tails occur therefore, it is clear if on X = 1, head comes then the process will be stopped and if tail comes then coin will be tossed second time. Clearly it will be repeated again and again till 5 tails come maximum

Then the value of X will be 1, 2, 3, 4, 5

S = {H,TH,TTH,TTTH or TTTTH,TTTTT}

Probability that heat comes in first throw

P(X = 1) = 1/2

P(X = 2) = P(TH) = P(T)P(H) = (1/2) x (1/2) = 1/4

P(X = 3) = P(TTH) = P(T)P(T)P(H) = (1/2) x (1/2) x (1/2) = 1/8

P (X = 4) = P(TTTH) = P(T)P(T)P(T)P(T)P(H)

= (1/2) x (1/2) x (1/2) x (1/2) = 1/16

P(X = 5) = P(TTTTH $\cup$ TTTTT)

= P(T)P(T)P(T)P(T)P(H) + P(T)P(T)P(T)P(T)P(T)



= (1/2) x (1/2) x (1/2) x (1/2) x (1/2) + (1/2) x (1/2) x (1/2) x (1/2) x (1/2)

= 1/16

Hence probability distribution will be like this

X = x	1	2	3	4	5
p(X)	1/2	1/4	1/8	1/16	1/16

Mean is given by

$$\sum_{i=1}^{5} p_i x_i = 1x\frac{1}{2} + 2x\frac{1}{4} + 3x\frac{1}{8} + 4x\frac{1}{16} + 5x\frac{1}{16} = 1.9$$

Example

A fair die is tossed. Let the random variable X denote the twice the number appearing on the die. Write the probability distribution of X. Calculate mean and variance.

Solution

Probability distribution of X

Probability distribution of X is obtained and tabulated as

x	2	4	6	8	10	12
p(x)	1/6	1/6	1/6	1/6	1/6	1/6

Mean

$$\mu = \sum xp(x) = 2\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right) + 8\left(\frac{1}{6}\right) + 10\left(\frac{1}{6}\right) + 12\left(\frac{1}{6}\right) = 7$$

Variance

$$\sigma^2 = \sum x^2 p(x) - \mu^2 = 4\left(\frac{1}{6}\right) + 16\left(\frac{1}{6}\right) + 36\left(\frac{1}{6}\right) + 64\left(\frac{1}{6}\right) + 100\left(\frac{1}{6}\right) + 144\left(\frac{1}{6}\right) - 7^2$$

= 11.67

Example (JNTUH 2009, 2010, 2019, 5 marks)

A sample of 4 items is selected at random from a box containing 12 items of which 5 are defective. Find the expected number of defective items.



Solution

Let X denote the number of defective items among 4 items drawn from 12 items.

Obviously X can take the values 0, 1,2, 3. or 4.

No. of good items = 7

No. of defective items = 5

Combination values are computed from following formula

$$^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

P(X = 0) = P(no defective) = $\frac{^{7}C_{4}}{^{12}C_{4}} = \frac{35}{495} = \frac{7}{99}$

P(X = 1) = P (one defective and 3 good items0

$$= \frac{^{7}C_{3}x^{5}C_{1}}{^{12}C_{4}} = \frac{175}{495} = \frac{35}{99}$$

P(X = 2) = P(two defective and 2 good items) = $\frac{^{7}C_{2}x^{5}C_{2}}{^{12}C_{4}} = \frac{42}{99}$

P (X = 3) = P(three defective and 1 good item) = $\frac{^{7}C_{1}x^{5}C_{3}}{^{12}C_{4}} = \frac{14}{99}$

p(X = 4) = P (all are defective) = $\frac{^{5}C_{4}}{^{12}C_{4}} = \frac{1}{99}$

PD is given below:

$X = x_i$	0	1	2	3	4
$P(X = x_i)$	7/99	35/99	42/99	14/99	1/99

Expected number of defective items

$$= E(X) = \sum x_i p(x_i) = 0\left(\frac{7}{99}\right) + 1\left(\frac{35}{99}\right) + 2\frac{42}{99} + 3\left(\frac{14}{99}\right) + 4.\left(\frac{1}{99}\right) = \frac{165}{99}$$

Problem

A sample of 3 items is selected at random from a box containing 10 items of which 4 are defective. Find the expected number of defective items.

Answer: 1.2



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Example (BPUT 2020, 6 marks)

A die is tossed thrice. A success is getting 1 or 6 on a toss, find the mean and variance of the number of successes.

Solution

Probability computations

We have n = 3. let X denote the number of success.

Probability of success = 2/6 = 1/3

Probability of failure = 1 - (1/3) = 2/3

$$P(X = 0) = P(\text{no success}) = P(\text{all failures}) = \frac{2}{3}.\frac{2}{3}.\frac{2}{3} = \frac{8}{27}$$

$$P(X = 1) = P(1 \text{ success, } 2 \text{ failures}) = \left({}^{3}C_{1}x\frac{1}{3}\right).\frac{2}{3}.\frac{2}{3} = \frac{4}{9}$$

$$P(X = 2) = P(2 \text{ success and } 1 \text{ failure}) = \left({}^{3}C_{2}x\frac{1}{3}\right).\frac{1}{3}.\frac{2}{3} = \frac{2}{9}$$

$$P(X = 3) = P(3 \text{ successes}) = \frac{1}{3}.\frac{1}{3}.\frac{1}{3} = \frac{1}{27}$$

Probability distribution

PD is given below:

x	0	1	2	3
P(x)	8/27	4/9	2/9	1/27

Mean

$$\mu = \sum x_i p(x_i) = 0\left(\frac{8}{27}\right) + 1\left(\frac{4}{9}\right) + 2\left(\frac{2}{9}\right) + 3\left(\frac{1}{27}\right) = 1$$

Variance

$$\sigma^2 = \sum x_i^2 p(x_i) - \mu^2 = 0^2\left(\frac{8}{27}\right) + 1^2\left(\frac{4}{9}\right) + 2^2\left(\frac{2}{9}\right) + 3^2\left(\frac{1}{27}\right) - 1^2 = \frac{2}{3}$$



CONTINUOUS PROBABILITY DISTRIBUTION

Probability Density Function

A continuous random variable has a probability zero of assuming exactly any of its values. The function f(x) is said to be probability density function of a continuous random variable X, if it is defined as

- $f(x) \geq 0, -\infty < x, \infty$
- $\int_{-\infty}^{\infty} f(x) = 1$
- $P(a < X < b) = \int_{a}^{b} f(x) = 1$

Continuous Distribution Function

If X is a continuous random variable, then the function f (x) is defined as:

$$F(x) = p(X \leq x) = \int_{-\infty}^{x} f(t)dt, -\infty < x < \infty$$

is called the continuous distribution function and f (x) is probability density function.

Properties of F(x)

- F(-∞) = 0
- f(∞) = 1
- 0 ≤ F(x) ≤ 1, -∞ < x < ∞
- P(a<X<b) = F(b) - F(a)
- $F'(x) = \frac{d}{dx} F(x) = f(x),\ f(x) \geq 0 \geq 0$

Example

If a random variable X has the probability distribution as follows:

$$f(x) = \begin{cases} 1/4 & -2 < x, 2 \\ 0 & otherwise \end{cases}$$

Compute P(X < 1) and (ii) P[(2X + 3) > 5]

Solution

(i) $P(X < 1) = \int_{-\infty}^{1} f(x)dx = \int_{-2}^{1} \frac{1}{4} dx = \frac{3}{4}$



(ii) $P[(2X+3)>5] = P[2x>5-3] = P[X>1] = \int_1^{\infty} f(x)dx = 1 - P(X<1)$

$= 1 - (3/4) = 1/4$

Example (JNTUH 2019, 5 marks)

Let the continuous random variable X have the probability density function,

$$f(x) = \begin{cases} 2/x^3 & if\ 1 < x < \infty \\ 0 & otherwise \end{cases}$$

Find F(x).

Solution

$$F(x) = \int_{-\infty}^{x} f(x)dx = \int_{-\infty}^{1} f(x)dx + \int_1^x f(x)dx = 0 + \int_1^x \frac{2}{x^3}dx$$

$$= 2\left|\frac{x^{-2}}{-2}\right|_1^x = -\left|\frac{1}{x^2}\right|_1^x = -\left(\frac{1}{x^2} - 1\right) = 1 - \frac{1}{x^2}$$

Example

Find the constant k such that

$$f(x) = \begin{cases} kx^2 & 0 < x < 3 \\ 0 & otherwise \end{cases}$$

is a probability function. Also, find the distribution function F(x) and P(1 < X ≤2).

Solution

Since f(x) is probability density function

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$\therefore$ $$\int_{-\infty}^{\infty} f(x)dx + \int_0^3 f(x)dx + \int_3^{\infty} f(x)dx = 1$$

$$0 + \int_0^3 kx^2 dx + 0 = 1$$

$\therefore$ k = 1/9

$\therefore$ $$f(x) = \begin{cases} x^2/9 & 0 < x < 3 \\ 0 & otherwise \end{cases}$$



$$F(x) = \int_{-\infty}^{x} f(x)dx = \int_{-\infty}^{0} f(x)dx + \int_{0}^{x} f(x)dx$$

$$= 0 + \int_{0}^{x} \frac{x^2}{9} dx = \frac{x^3}{27}$$

Now $$F(x) = \begin{cases} x^3/27 & 0 < x < 3 \\ 0 & otherwise \end{cases}$$

$$p(1 < x \le 2) = \int_{1}^{2} f(x)dx = \int_{1}^{2} \frac{x^2}{9} dx = \frac{7}{27}$$

Problem

A function is defined as

$$f(x) = \begin{cases} 0 & x < 2 \\ \frac{2x+3}{18} & 2 \le x \le 4 \\ 0 & x > 4 \end{cases}$$

Show that f(x) is a probability density function and find P(2 < X < 3).

Answer: 4/9

MEAN, MEDIAN, MODE, VARIANCE AND STANDARD DEVIATION FOR CONTINUOUS PROBABILITY DISTRIBUTION

Mean

The mean or average value (μ) of the probability distribution of a continuous random variable X is called the expectation and is denoted by E(X).

$$\mu = E(X) = \int_{-\infty}^{\infty} x(f)dx$$

Median

The median is the point which divides the entire distribution into two equal parts. In case of a continuous distribution, the median is the point which divides the total area into two equal parts. Thus, if a continuous random variable X is defined from a to b and M is the median,

$$\int_{a}^{M} f(x)dx = \int_{M}^{b} f(x)dx = \frac{1}{2}$$



Mode

The mode is value of x for which f (x) is maximum. Mode is given by

f'(x) = 0 and f''(x) < 0 for a < x < b

Variance

The variance of the probability distribution of a continuous random variable X is given by

$$Var(X) = \sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx = \int_{-\infty}^{\infty} x^2 f(x)dx - \mu^2$$

Standard deviation

The standard deviation of the probability distribution of a continuous random variable X is given by

$$SD = \sqrt{\text{var}(X)} = \sigma$$

Example

For the continuous random variable having pdf

$$f(x) = \begin{cases} 4x^3 & 0 \le x \le 1 \\ 0 & otherwise \end{cases}$$

Find the mean and variance of X.

Solution

Mean

$$\mu = \int_{-\infty}^{\infty} xf(x)dx = \int_{-\infty}^{0} xf(x)dx + \int_{0}^{1} xf(x)dx + \int_{1}^{\infty} xf(x)dx$$

$$= 0 + \int_0^1 x(4x^3)dx + 0 = \frac{4}{5}$$

Variance of X

$$Var(x) = \int_{-\infty}^{\infty} x^2 f(x)dx - \mu^2$$

$$= \int_{-\infty}^{0} x^2 f(x)dx + \int_{0}^{1} x^2 f(x)dx + \int_{1}^{\infty} x^2 f(x)dx - \mu^2$$

$$= 0 + \int_0^1 x^2(4x^3)dx + 0 - \left(\frac{4}{5}\right)^2 = \frac{2}{75}$$



Problem

For the triangular distribution

$$f(x) = \begin{cases} x & 0 < x \le 1 \\ 2 - x & 1 \le x \le 2 \\ 0 & otherwise \end{cases}$$

Find the mean and variance.

Answer: 1, 1/6

BINOMIAL DISTRIBUTION

Consider n independent trials of a random experiments which results in either success or failure. Let p be the probability of success remaining constant every time and q = 1 – p be the probability of failure. The probability of **x successes** and **n – x failures** is given by $p^x\, q^{n-x}$. But these x successes and n – x failures can occur in any of the ${}^nC_x$ ways in each of which the probability is same.

Hence, the probability of x successes is

$$P(X = x) = {}^nC_x p^x q^{n-x},\ x = 0, 1, 2 \ldots n$$

Examples of binomial distribution

An example Binomial Distribution problem in the world of engineering may read something like this:

A sample containing 400 items is taken from the output of a production line. Defective items occur randomly and independently at a rate of 1.6% of the population. The probability that the sample will have no more than 2.5% defective items is most close to what?

The first step to solving this problem is knowing quickly that it is indeed a Binomial Distribution problem.

Let's read over the problem statement again.

- Does this scenario involve a **fixed** number of trials? Yes, 400 items
- Does the experiment produce **only binary results**? Yes, either a defective or non-defective item.
- Is the **probability** of success in each trial **constant**? Yes, 1.6% defect rate

The core characteristics of a Binomial experiment are present.

A few examples:



- Number of defective bolts in a box containing n bolts.
- Number of post-graduates in a group of n people.
- Number of oil wells yielding natural gas in a group of n wells test drilled.
- Number of machines lying idle in a factory having n machines.

Mean of the binomial distribution

$$E(X) = \sum xp(x) = \sum_{x=0}^{n} x^{n}C_{x}p^{x}q^{n-x}$$

$$= 0.^{0}C_{0}p^{0}q^{n} + 1.^{n}C_{1}pq^{n-1} + 2.^{n}C_{2}p^{2}q^{n-2} + ... + np^{n}$$

$$= np\left[q^{n-1} + {}^{(n-1)}C_{1}q^{n-2}p + {}^{(n-1)}C_{2}q^{n-3}p^{2} + ...p^{n-1}\right]$$

$$= np(q+p)^{n-1}$$

= np [As p + q = 1]

Variance of the binomial distribution

$$Var(X) = E(X^{2}) - \mu^{2}$$

$$= \sum_{x=0}^{n} x^{2}p(x) - \mu^{2}$$

$$= \sum_{x=0}^{n}\left[x + x(x-1)\right]^{n}C_{x}p^{x}q^{n-x} - \mu^{2}$$

$$= \sum_{x=0}^{n} x^{n}C_{x}p^{x}q^{n-x} + \sum_{x=0}^{n} x(x-1)^{n}C_{x}p^{x}q^{n-x} - \mu^{2}$$

$$= np + \sum_{x=0}^{n} x(x-1)\frac{n(n-1)}{x(x-1)}.{}^{(n-2)}C_{x-2}p^{x-2}q^{n-x} - \mu^{2}$$

$$= np + n(n-1)p^{2}\sum_{x=0}^{n} {}^{(n-2)}C_{(x-2)}p^{x-2}q^{n-x} - \mu^{2}$$

$$= np + n(n-1)p^{2}.(q+p)^{n-2} - \mu^{2}$$

$$= np + n(n-1)p^{2} - \mu^{2}$$ [as p + q = 1]

$= np[1 + (n - 1)p] - \mu^{2}$

$= np[1 - p + np] - \mu^{2}$

$= np[q = np] - \mu^{2}$ [1 - p = q]



$= np[q + np] - (np)^2$

$= npq$

Standard deviation of the binomial distribution

$$SD = \sqrt{Variance} = \sqrt{npq}$$

Recurrence relation for the binomial distribution

$$P(X = x+1) = \frac{n-x}{x+1}.\frac{p}{q}.P(X = x)$$

Example (BPUT 2020, 2 marks)

A fair coin is tossed 400 times. If X is the number of heads obtained, find the expected value and variance of X.

Solution

We have a Binomial distribution,

n = 400

p = 0.5 (success)

q =1− p=1− 0.5 = 0.5 (failure)

For a Binomial distribution the expected number of successes (a head obtained) and the variance are given by the formulas:

Mean (expected value)

$\mu = np = 400 \times 0.5 = 200$

Variance

$\sigma^2 = npq = 400 \times 0.5^2 = 100$

Example

If 10% of the screws produced by a machine are defective, find the probability that out of 5 screws chosen at random, (i) none is defective, (ii) one is defective, and (iii) at most two are defective.

Solution

Let p be the probability of defective screws.

p = 0.1



q = 1 – p = 1 – 0.1 = 0.9,

n = 5

Probability that x screws out of 5 screws are defective

$$P(X = x) = {}^{n}C_x p^x q^{n-x} = {}^{5}C_x (0.1)^x (0.9)^{5-x}; x = 0,1,2..5$$

Probability that none of the screws out of 5 screws is defective

$$P(X = 0) = {}^{5}C_0 (0.1)^0 (0.9)^5 = 0.5905$$

Probability that one screw out of 5 screws is defective

$$P(X = 1) = {}^{5}C_1 (0.1)^1 (0.9)^4 = 0.3281$$

Probability that at most 2 screws out of 5 screws are defective

$$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$= \sum_{x=0}^{2} P(X = x) = \sum_{x=0}^{2} {}^{5}C_x (0.1)^x (0.9)^{5-x} = 0.9914$$

Example (GTU NM 2020, 4 marks)

Out of 800 families with 4 children each how many families would be expected to have (i) at least one boy (ii) 2 boys and 2 girls Assume equal probabilities for boys and girls.

Solution

Since probabilities for boys and girls are equal

p = probability of having a boy = 1/2

q = probability of having a girl (or not having boy) = 1/2

n = 4; N = 800

The expected number of families having 2 boys and 2 girls

$$= 800 {}^{4}C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 = 800x6x\frac{1}{16} = 300$$

The expected number of families having at least one boy

$$= 800\left[{}^{4}C_1 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right) + {}^{4}C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 + {}^{4}C_3 \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^3 + {}^{4}C_4 \left(\frac{1}{2}\right)^4\right]$$

= 750



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Example (UTU 2017, 5 marks)

Six dice are thrown 729 times how many do you expect at least three dice to show a five or six.

Solution

p = chance of getting 5 or 6 with one die = 2/6 = 1/3

q = 1 - p = 1 - 1/3 = 2/3

n = 6; N = 729

Since dice are in sets of 6 and there are 729 sets.

The binomial distribution is

$$N(q+p)^n = 729\left(\frac{2}{3}+\frac{1}{3}\right)^6$$

The expected number of times al least three dice showing five or six.

$$= 729\left[{}^6C_3\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^3 + {}^6C_4\left(\frac{2}{3}\right)^2\left(\frac{1}{4}\right)^4 + {}^6C_5\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^5 + {}^6C_6\left(\frac{1}{3}\right)^6\right]$$

Problem (JNTU 2010)

A coin is biased in a way that a head is twice as likely to occur as a tail. If the coin is tossed 3 times, find the probability of getting 2 tail and 1 head.

Answer: 2/9

Problem (JNTUH 2010)

The probability that a man hitting a target is 1/3. If he fires 6 times, find the probability that he fires (i) At the most 5 times (ii) Exactly once (iii) At least two times.

Answer: 0.9986, : 0.2634, : 0.6488

Example (JNTUH 2015)

Fit a binomial distribution to the following data

x	0	1	2	3	4
y	38	144	342	287	164

Solution

Here n = number of trials = 4

Total frequency



$$N = \sum_{i-0}^{4} f_i = 38 + 144 + 342 + 287 + 164 = 975$$

Mean of binomial distribution is

$$\mu = np = \frac{\sum f_i x_i}{\sum f_i} = \frac{0(38) + 1(144) + 2(342) + 3(287) + 4(164)}{975} = 2.4051$$

np = 4p

Putting values p = 2.4051/4 = 0.6

So q = 1 - p = 1 - 0.6 = 0.4

Thus the binomial distribution that fits the given data is

$$b(x : n : p) = b(x; 4; 0.6) = {}^{n}C_x p^x q^{n-x} = {}^{4}C_x (0.6)^x (0.4)^{4-x}$$

$$\therefore \quad p(x) = {}^{4}C_x (0.6)^x (0.4)^{4-x}$$

The theoretical frequencies are

x	0	1	2	3	4
P(x)	0.0256	0.1536	0.3456	0.3456	0.1296
Expected frequency (rounded) = N x P(x) = 975P(x)	25	150	337	337	126

Problem (JNTU 2004)

Fit the binomial distribution to the following data

x	*0*	*1*	*2*	*3*	*4*	*5*
f	*38*	*144*	*342*	*287*	*164*	*25*

Answer: 33, 162, 316, 309, 151. 29

POISSON DISTRIBUTION

Poisson distribution is a limiting case of binomial distribution under the following conditions:

- The number of trials should be infinitely large, i.e., $n \to \infty$
- The probability of successes p for each trial should be very small, i.e., $p \to 0$.
- $np = \lambda$ should be finite where A is a constant. where λ is called the *parameter of the distribution*.

The binomial distribution is



$$P(X=x) = {}^{n}C_x p^x q^{n-x} = {}^{n}C_x \left(\frac{p}{q}\right)^x q^n = {}^{n}C_x \left(\frac{p}{1-p}\right)^x (1-p)^n$$

Putting $p = \lambda/n$

$$P(X=x) = \frac{n(n-1)(n-2)....(n-x+1)}{x!}\left(\frac{\lambda/n}{1-(\lambda/n)}\right)^x \left(1-\frac{\lambda}{x}\right)^n$$

$$= \frac{n(n-1)(n-2)....(n-x+1)}{x!}\cdot\frac{\lambda^x}{n^x}\cdot\frac{1}{(1-(\lambda/n))^x}\left(1-\frac{\lambda}{x}\right)^n$$

$$= \frac{n(n-1)(n-2)....(n-x+1)}{x!}\cdot\frac{\lambda^x}{n^x}\cdot\left(1-\frac{\lambda}{x}\right)^{n-x}$$

$$= \frac{1\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)....\left[1-\left(\frac{x-1}{n}\right)\right]}{x!}\lambda^x\cdot\left(1-\frac{\lambda}{n}\right)^{n-x}$$

Since $\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{n-x} = e^{-\lambda}$

and $\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{n-x} = \lim_{n\to\infty}\left(1-\frac{2}{n}\right) = 1$

Taking the limits of both the sides as $n \to \infty$

$$P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!},\ x = 0,1,2...\infty$$

Examples of Poisson distribution

- Number of defective bulbs produced by a reputed company
- Number of telephone calls per minute at a switchboard
- Number of cars passing a certain point in one minute
- Number of printing mistakes per page in a large text
- Number of persons born blind per year in a large city
- Emission of radioactive particles.
- Number of suicides or deaths due to heart attack in one minute.



Mean of the Poisson distribution

$$E(X)=\sum_{x=0}^{\infty}xp(x)=\sum_{x=0}^{\infty}x.\frac{e^{-\lambda}\lambda^{x}}{x!}=\sum_{x=0}^{\infty}\frac{xe^{-\lambda}\lambda\lambda^{x-1}}{x!}$$

$$=e^{-\lambda}.\lambda\sum_{x=1}^{\infty}\frac{\lambda^{x-1}}{(x-1)!}=\lambda e^{-\lambda}\left(1+\lambda+\frac{\lambda^{2}}{2!}+...\right)=\lambda e^{-\lambda}e^{\lambda}=\lambda$$

Variance of the Poisson distribution

$$Var(X)=E(X^{2})-\mu^{2}=\sum_{x=0}^{\infty}x^{2}p(x)-\mu^{2}$$

$$=\sum_{x=0}^{\infty}x^{2}.\frac{e^{-\lambda}\lambda^{x}}{x!}-\lambda^{2}=\sum_{x=0}^{\infty}x[(x-1)+x].\frac{e^{-\lambda}\lambda^{x}}{x!}-\lambda^{2}$$

$$=\sum_{x=0}^{\infty}\frac{x(x-1)e^{-\lambda}\lambda^{x}}{x!}+\sum_{x=0}^{\infty}\frac{xe^{-\lambda}\lambda^{x}}{x!}-\lambda^{2}$$

$$=\sum_{x=0}^{\infty}\frac{x(x-1)e^{-\lambda}\lambda^{x-2}\lambda^{2}}{x(x-1)(x-2)...1}+\lambda-\lambda^{2}=e^{-\lambda}\lambda^{2}\sum_{x=2}^{\infty}\frac{\lambda^{x-2}}{(x-2)!}+\lambda-\lambda^{2}$$

$$=e^{-\lambda}\lambda^{2}\left(1+\lambda+\frac{\lambda^{2}}{2!}+...\right)+\lambda-\lambda^{2}$$

$$=\lambda$$

Standard deviation of the Poisson distribution

$$SD=\sqrt{Variance}=\sqrt{\lambda}$$

Recurrence relation for the Poisson distribution

$$p(x+1)=\frac{\lambda}{x+1}p(x)$$

Example

A book of 585 pages contains 43 typographical errors. If these errors are randomly distributed throughout the book, what is the probability that 10 pages, selected at random, will be free from errors?

Solution

Let p be the probability of errors in a page.



P = 43/585 = 0.0735

And n = 10

Since

- p is very small and
- n is large,

Poisson distribution will be used.

$\lambda = np = 10(0.0735) = 0.735$

Let X be the random variable which denotes the errors in the pages.

Probability of x errors in a page in a book of 585 pages

$$p(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-0.735}(0.735)^x}{x!}; x = 0,1,2...$$

Probability that a random sample of 10 pages will contain no error.

$$p(X = 0) = \frac{e^{-0.735}(0.735)^0}{0!} = 0.4795$$

Example

Suppose that a book of 600 pages contains 40 printing mistakes. Assume that these errors are randomly distributed throughout the book. The number of errors per page has a Poisson distribution. What is the probability that 10 pages selected at random will be free of errors?

Answer: 0.51

Example

Example (UTU 2016, 10 marks)

If the variance of Poisson distribution is 2, find the probability for r = 1, 2, 3, 4 from the recurrence relation of the Poisson distribution. Also find p(r ≥ 4).

Solution

Here $\lambda = 2$

$\therefore \quad P(x+1) = \frac{2}{x+1}P(x)$

If x = 0 $\quad P(0+1) = P(1) = \frac{2}{0+1}(0.1353) = 0.2706$ $\quad [P(0) = e^{-\lambda} = e^{-2} = 0.1353]$



If x = 1 $P(2) = \frac{2}{1+1} P(1) = \frac{2}{2}(0.2706) = 0.2706$

If x = 2 $P(3) = \frac{2}{2+1} P(2) = \frac{2}{3}(0.2706) = 0.1804$

If x = 2 $P(4) = \frac{2}{3+1} P(3) = \frac{1}{2}(0.1804) = 0.0902$

Now $P(r \geq 4) = P(4) + P(5) + P(6) + \ldots$

$$= 1 - [P(0) + P(1) + P(2) + P(3)]$$
$$= 1 - [0.1353 + 0.2706 + 0.2706 + 0.1804] = 0.1431$$

Example (UTU 2015, 10 marks)

The arrival rate of the customers arriving at a bank follows Poisson Distribution with a mean arrival rate of 4 per 10 minutes interval. Find the probability that

(i) Exactly 0 customer will arrive in 10 minutes interval

(ii) Exactly 2 customers will arrive in 10 minutes interval

(iii) At most 2 customers will arrive in 10 minutes interval

(iv) At least 2 customers will arrive in 10 minutes interval

Solution

Given $\lambda = 4$

Now $P(x = 0) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-4}4^0}{0!} = 0.018$

$$P(x = 2) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-4}4^2}{2!} = 0.146$$

$$P(x \leq 2) = \sum_{x=0}^{2} \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-4}4^0}{0!} + \frac{e^{-4}4^1}{1!} + \frac{e^{-4}4^2}{2!} = 0.238$$

$$P(x \geq 3) = 1 - \sum_{x=0}^{2} \frac{e^{-\lambda}\lambda^x}{x!} = 1 - \left(\frac{e^{-4}4^0}{0!} + \frac{e^{-4}4^1}{1!} + \frac{e^{-4}4^2}{2!} \right) = 0.762$$

Problem

Six coins are tossed 6,400 times. Using the Poisson distribution, what is the appropriate probability of getting 6 heads 10 times.



Answer: 1.025 x 10^{-30}

Problem

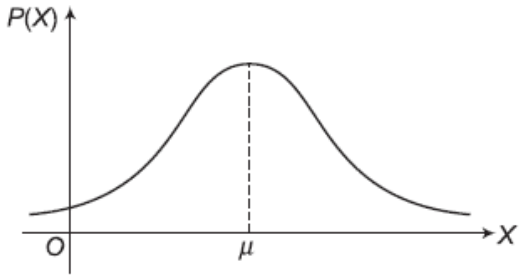
Suppose 3% of bolts made by a machine are defective, the defects occurring at random during production. If bolts are packaged 50 per box, find Poisson approximation to it, that a given box will contain 5 defectives.

Answer: 0.01412

NORMAL DISTRIBUTION

A continuous random variable X is said to follow normal distribution with mean μ and variance σ^2, if its probability function is given by

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2};-\infty<X\infty,-\infty<\mu<\infty,\sigma>0$$



where μ and σ are called **parameters of the normal distribution**. The curve representing the normal distribution is called the **normal curve**.

Mean of the normal distribution

$$E(X)=\int_{-\infty}^{\infty}xf(x)dx=\int_{-\infty}^{\infty}x.\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}dx$$

Putting $\frac{x-\mu}{\sigma}=t;dx=\sigma dt$

$$E(X)=\int_{-\infty}^{\infty}(\mu+\sigma t).\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}t^2}dt$$

$$=\mu\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}t^2}dt+\int_{-\infty}^{\infty}\sigma\frac{t}{\sqrt{2\pi}}e^{-\frac{1}{2}t^2}dt$$

Putting $t^2 = u$ in the second integral.

We get 2tdt = du

When $t \to \infty$, $u \to \infty$

When $t \to -\infty$, $u \to \infty$

$$E(X) = \mu \frac{1}{\sqrt{2\pi}} . \sqrt{2\pi} + \int_{\infty}^{\infty} \sigma . \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u} \frac{du}{2} \; ; \; \because \int_{-\infty}^{\infty} e^{-\frac{1}{2}t^2} dt = \sqrt{2\pi}$$

$= \mu + 0$ [because the limits of integration are same]

$= \mu$

Variance of the normal distribution

$$Var(X) = E(X - \mu^2)$$

$$= \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx = \int_{-\infty}^{\infty} (x-\mu)^2 \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Putting $(x - \mu)/\sigma = t$, $dx = \sigma dt$

$$Var(X) = \int_{-\infty}^{\infty} \sigma^2 t^2 . \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} dt = \frac{\sigma^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} t^2 e^{-\frac{1}{2}t^2} dt = \frac{2\sigma^2}{\sqrt{2\pi}} \int_{0}^{\infty} t^2 e^{-\frac{1}{2}t^2} dt$$

[because integral is an even function]

Putting $t^2/2 = u$; $t = \sqrt{2u}$

$$dt = \sqrt{2} . \frac{1}{2\sqrt{u}} du = \frac{1}{\sqrt{2u}} du$$

When t = 0, u = 0

When $t = \infty$, $u = \infty$

$$Var(X) = \frac{2\sigma^2}{\sqrt{2\pi}} \int_{0}^{\infty} 2ue^{-u} . \frac{1}{\sqrt{2u}} du$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \int_{0}^{\infty} e^{-u} u^{1/2} du$$

$= \sigma^2$

Standard deviation of the normal distribution

SD = σ



Standard normal (Z) table, area between 0 and z

A sample of normal table is given below:

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517

Example

If X is a normal variate with a mean of 30 and an SD of 5, find the probabilities that (i) 26 £ X £ 40, and (ii) $X \geq 45$.

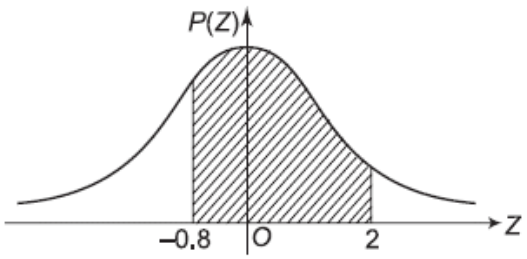
Solution

$$\mu = 30, \sigma = 5$$

$$Z = \frac{X - \mu}{\sigma}$$

(i) When $X = 26, Z = \frac{26-30}{5} = -0.8$

When $X = 40, Z = \frac{40-30}{5} = 2$



$P(26 \leq X \leq 40) = P(-0.8 \leq Z \leq 2)$ [see figure]

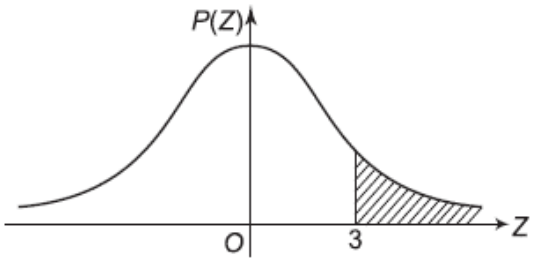
$= P(-0.8 \leq Z \leq 0) + P(0 \leq Z \leq 2)$

$= P(0 \leq Z \leq 0.8) + P(0 \leq Z \leq 2)$ [by symmetry]

= 0.2881 + 0.4772 [from normal table]

= 0.7653

(ii) When $X = 45, Z = \frac{45-30}{5} = 3$



$P(X \geq 45) = P(Z \geq 3)$ [see figure]

$= 0.5 - P(0 < Z < 3)$

$= 0.5 - 0.4987$

$= 0.0013$

Problem

X is normally distributed and the mean of X is 12 and the SD is 4. Find out the probability of the following: (i) $X \geq 20$ (ii) $X \leq 20$ (iii) $0 \leq X \leq 12$.

Answer: 0.0228; 0.9772; 0.4987

Example (JNTUH 2019, 5 marks)

In a normal distribution, 7% of the items are under 35 and 89% are under 63. Determine the mean and variance of the distribution.

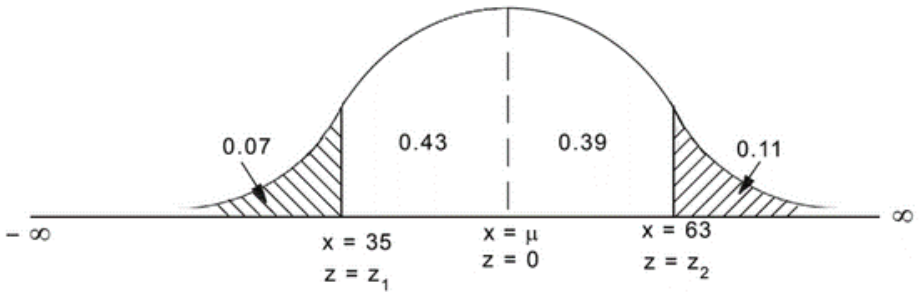
Solution

Let μ be the mean (at z = 0) and σ the standard deviation of the normal curve. 7% of the times are under 35 means the area to the left of the ordinate x = 35.

Given P(X < 35) = 0.07 and P(X < 63) = 0.89

$\therefore$ P(X > 63) = 1 - P(X < 63) = 1 - 0.89 = 0.11

The points X = 35 and X = 63 are shown in the following figure.



When $x = 35, z = \frac{x-\mu}{\sigma} = \frac{35-\mu}{\sigma} = -z_1 (say)$ (1)

When $x = 63, z = \frac{x-\mu}{\sigma} = \frac{63-\mu}{\sigma} = z_2 (say)$ (2)

From the figure, we have

$P(0 < z < z_2) = 0.39$ which gives $z_2 = 1.23$ (from normal tables)

$P(0 < z < z_1) = 0.43$ which gives $z_1 = 1.48$

From (1), we have

$$\frac{35-\mu}{\sigma} = -1.48 \quad (3)$$

From (2), we have

$$\frac{63-\mu}{\sigma} = -1.23 \quad (4)$$

Solving these equations,

$\sigma = 10.332$, $\mu = 50.3$

Variance = $\sigma^2 = 106.75$

Example (UTU 2017, 5 marks)

The mean height of 500 students is 151 cm and the standard deviation is 15 cm. Assuming that the heights are normally distributed, find how many students have heights between 120 cm and 155 cm.

Solution

We have

$N = 500$, $\mu = 151$, $\sigma = 15$

If x = 120, then

$$Z = \frac{120-\mu}{\sigma} = \frac{120-151}{15} = -2.07$$

If x = 155, then

$$Z = \frac{155-151}{15} = 0.27$$

$\therefore \quad P(120 \le x \le 150) = P(-2.07 \le Z \le 0.27)$



$= P(-2.07 \le Z \le 0) + P(0 \le Z \le 0.27)$

$= P(0 \le Z \le 2.07) + P(0 \le Z \le 0.27)$ [by symmetry]

$= 0.4808 + 0.1064$

$= 294$

Example (JNTUH 2018, 5 marks)

The mean voltage of a battery is 15 and standard deviation 0.2. Find the probability that four such batteries connected in series will have a combined voltage of 60.8 or more volts.

Solution

Let, mean voltage of a batteries 1,2,3,4 be $\overline{x_1}, \overline{x_2}, \overline{x_3}, \overline{x_4}$ the mean of the series of the four batteries connected is

$\mu(\overline{x_1} + \overline{x_2} + \overline{x_3} + \overline{x_4})$

$= \mu(\overline{x_1}) + \mu(\overline{x_2}) + \mu(\overline{x_3}) + \mu(\overline{x_4})$

$= 15 + 15 + 15 + 15 = 60$

$\sigma(\overline{x_1} + \overline{x_2} + \overline{x_3} + \overline{x_4})$

$= \sqrt{\sigma^2(\overline{x_1}) + \sigma^2(\overline{x_2}) + \sigma^2(\overline{x_3}) + \sigma^2(\overline{x_4})}$

$= \sqrt{4(0.2)^2} = 0.4$

Let X be the combined voltage of the series.

When x = 60.8, $z = \frac{x - \mu}{\sigma} = \frac{60.8 - 60}{0.4} = 2$

Then the probability that the combined voltage is more than 60.8 is given by $P(X \ge 60.8) = P(Z \ge 2) = 0.0228$.

Example (JNTU 2007, 2010, 2012)

The marks obtained in mathematics by 1000 students is normally distributed with mean 78% and standard deviation 11%. Determine

(i) How many students got marks above 90%

(ii) What was the highest mark obtained by the lowest 10% of the students

Solution

(i) When x = 0.9

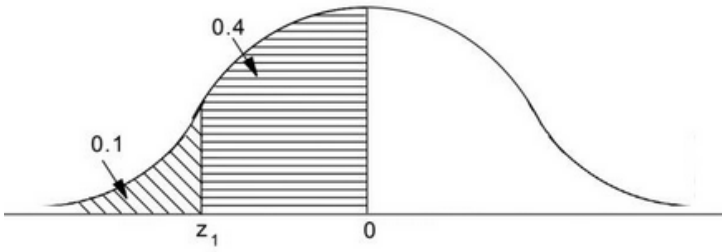


$$z = \frac{x-\mu}{\sigma} = \frac{0.9-0.78}{0.11} = 1.09 = z_1(say)$$

Hence the number of students with marks more than 90%

= 0.1379 x 1000 = 138

(ii) The 0.1 area to the left of z corresponds to the lowest 10% of the students.



From figure

0.4 = 0.5 - 0.1 = 0.5 - area from 0 to z_1

$\therefore$ z_1 = -1.28 (from tables)

Thus $-1.28 = \frac{x-\mu}{\sigma} = \frac{x-0.78}{0.11}$

$\therefore$ x = 0.6392

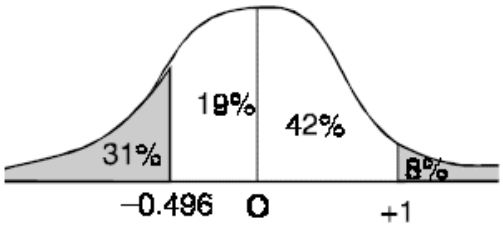
Hence the highest marks obtained by the lowest 10% of students = 64%

Example (AKTU 2019, UTU 2014)

In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. It is given that if $\frac{1}{\sqrt{2\pi}}\int_0^t e^{-x^2/2}dx$ *then f(04965) = 0.19 and f(1.405) = 0.42.*

Solution

Let μ be the mean and σ the SD.



If x = 45 $z = \frac{45 - \mu}{\sigma}$

If x = 64 $z = \frac{64 - \mu}{\sigma}$

The area between 0 and $(45 - \mu)/\sigma = 0.50 - 0.31 = 0.19$.

Now, from given data for area 0.19, z = 0.496

$$\frac{45 - \mu}{\sigma} = -0.496 \quad (1)$$

Now, area between z = 0 and $z = (64 - \mu)/\sigma = 0.5 - 0.08 = 0.42$

From given data for area 0.42, z = 1.405

$$\frac{64 - \mu}{\sigma} = 1.405 \quad (2)$$

Solving (1) and (2)

$\mu = 50$ and $\sigma = 10$.

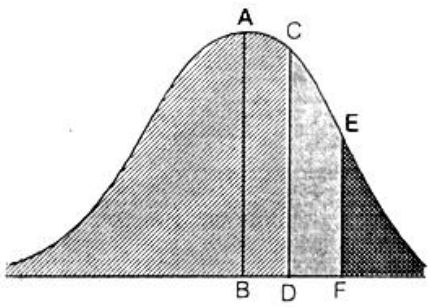
Example (GTU 2020)

If Skulls are classified as A, B, C according as the length and breadth index as under 75, between 75 and 80 or over 80; find the approximately mean and the standard deviation of the classes in which A are 58%, B are 38% and C are 4%. Given $f(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-t^2/2} dt$ *, f(0.20) = 0.08 and f(1.75) = 0.46.*

Solution

Let us assume that μ is the mean and σ is the standard deviation of the distribution.

As the total frequency is one, the frequency of skull A, whose length and breadth index is under 75 is 0.58, the frequency of skull B whose index lies between 75 and 80 is 0.38 and the frequency of skull C, whose index is over 80 is 0.04.



From figure, we see that the total area to the left of ordinate CD is 0.58, area between ordinates CD and EF is 0.38 and the area to the right of ordinate EF is 0.04.

Thus, area between origin and X= 75 - μ i.e.t the area ABCD = 0.08.

Transforming it to standard normal variate the corresponding area to

$$z = \frac{75-\mu}{\sigma} = 0.08$$

But, we are given f(0.20) = 0.08

$$\therefore \quad \frac{75-\mu}{\sigma} = 0.20 \qquad (1)$$

Similarly, the area between origin and X = 80 - μ i.e. area ABFE

= 0.08 + 0.38 = 0.46

i.e. corresponding area to

$$z = \frac{80-\mu}{\sigma} = 0.46$$

But, we are given f(1.75) = 0.46

$$\therefore \quad \frac{80-\mu}{\sigma} = 1.75 \qquad (2)$$

Solving (1) and (2)

μ = 74.3 and σ = 3.23

Example (JNTUH 2003, 2018, 5 marks)

The diameter of motor shafts in a lot has a mean of 0.249 inch and a S.D of 0.003 inch. The inner diameter of bearings in another lot have a mean of 0.255 inch and a S.D of 0.002 inch.

(i) What arc the mean and the S.D of the clearances between shafts and bearings selected from those lots ?

(ii) If a shaft and a bearing are selected at random, what is the probability that the shaft will not fit inside, the bearing ? Assume that both dimensions arc normally distributed.

Solution

Let x_1' be the mean diameter of the bearing and x_2' be the mean diameter of the shaft.

Given that $x_1' = 0.255$; $x_2' = 0.249$; $\sigma_1 = 0.002$, $\sigma_2 = 0.003$

(i) Let x_d' be the mean diameter of the difference in the two diameters.



$x_d' = x_1' - x_2' = \mu_1 - \mu_2 = 0.255 - 0.249 = 0.006$

$$\sigma_d = \sqrt{\sigma_1^2 + \sigma_2^2} = \sqrt{(0.002)^2 = (0.003)^2} = 0.0036$$

(ii) Shaft will not fit inside bearing if $d < 0$.

Now, $$z = \frac{0 - 0.006}{0.0036} = -1.6666$$

The probability that the shaft will not fit inside bearing

$= P(d < 0) = P(z < -1.6666)$

$= 0.5 - 0.4515 = 0.0485$

Problem

In a normal distribution. 7% of the items are under 35 and 89% are under 63. Determine mean and variance of distribution. [Area of z for 0.43 = 1.48. Area of z for 0.39 = 1.23]

Ans. $\mu = 50.29$. $\sigma^2 = 106.73$

Problem

Assume that the mean height of soldiers is 68.22 inches with - 10.8. How many soldiers in a regiment $\sigma^2 = 1000$ would you expect to be over 6 feet tall ? Given that the area under the standard normal curve between z = 0 and z = 0.35 is 0.1368 and between z = 0 and z = 1.15 is 0.3746.

Answer: 125

Problem

The lifetime of a certain kind of batteries has a mean life of 400 hours and the standard deviation as 45 hours. Assuming the distribution of lifetime to be normal, find (i) the percentage of batteries with a lifetime of at least 470 hours, (ii) the proportion of batteries with a lifetime between 385 and 415 hours, and (iii) the minimum life of the best 5% of batteries.

Answer: 5.94%, 25.86%, 474.25 hours



ASSIGNMENT

Q.1. (GTU NM 2020, 3 marks): A can hit a target 4 times in 5 shots, B, 3 times in 4 shots and C, 2 times in 3 shots. They fire a volley (it means one shot each). Find the probability that the target will be hit.

Answer: Solved in this module.

Q.2. (AKTU 2019, 3.5 marks): If the probability of hitting a target is 10% and 10 shots are fired independently, what is the probability that the target will be hit at least once?

Answer: p = 10/100 = 1/10; q = 1 - p = 9/10; probability that the target will be hit at least once = 1 - $({}^{10}C_0 p^0 q^{10})$.

Answer is 0.6513.

Q.3. (AKTU 2018, 2020, 2 marks): Define the Binomial distribution with mean and variance.

Q.4. (BPUT 2020, 2 marks): Distinguish between Binomial distribution and Normal distribution.

Q.5. (JNTUH 2019, 5 marks): A discrete random variable X has the mean 6 and variance 2. If it is assumed that the distribution is binomial, find the probability that $5 \leq x \leq 7$.

Answer: 4672/6561

Q.6. (UTU 2018, 10 marks): Define Binomial probability distribution. Fit the Binomial probability distribution to the following frequency data:

x	0	1	2	3
y	28	62	10	4

Answer: Similar problem is solved in this module.

Q.7. (AKTU 2021, 2 marks): Define the Poisson distribution with mean and variance.

Q.8. (AKTU 2021, 7 marks): Show that Poisson distribution is a limiting case of Binomial distribution when p is very small and μ is very large. Also find the mean and variance of the Poisson distribution.

Q.9. (UTU 2015, 10 marks): The arrival rate of the customers arriving at a bank follows Poisson Distribution with a mean arrival rate of 4 per 10 minutes interval. Find the probability that

(i) Exactly 0 customer will arrive in 10 minutes interval

(ii) Exactly 2 customers will arrive in 10 minutes interval

(iii) At most 2 customers will arrive in 10 minutes interval

(iv) At least 2 customers will arrive in 10 minutes interval

Answer: Solved in this module.

Q.10. (UTU 2016, 10 marks): If the variance of Poisson distribution is 2, find the probability for r = 1, 2, 3, 4 from the recurrence relation of the Poisson distribution. Also find $p(r \geq 4)$.

Answer: Solved in this module.

Q.11. (UTU 2013, 10 marks): A manufacturer knows that the condensers he makes contains on an average 1 % of defectives. He packs them in boxes of 100. What is the probability that a box picked at random will contain 4 or more faulty condensers?

Answer: 0.019; Poisson distribution.

Q.12. (AKTU 2018, 2020, 2021, 7 marks): Find the mean and variance of normal distribution.



Q.13. (UTU 2014, AKTU 2019): In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. It is given that if $f(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-\frac{1}{2}x^2} dx$ then f(0.5) = 0.19, f(1.4) = 0.42.

Answer: Solved in this module.

Q.14. (GTU 2020): If Skulls are classified as A, B, C according as the length and breadth index as under 75, between 75 and 80 or over 80; find the approximately mean and the standard deviation of the classes in which A are 50%, B are 38% and C are 40%. Given $f(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-t^2/2} dt$, f(0.20) = 0.08 and f(1.75) = 0.46.

Answer: Solved in this module.

Q.15. (JNTUH 2019, 3 marks): Define moment generating function of a random variable.

Q.16. (JNTUH 2019, 5 marks): Let the continuous random variable X have the probability density function,

$$f(x) = \begin{cases} 2/x^3 & if\ 1 < x < \infty \\ 0 & otherwise \end{cases}$$

Find F(x).

Answer: Solved in this module.

Q.17. (BPUT 2020, 2 marks): A fair coin is tossed 400 times. If X is the number of heads obtained, find the expected value and variance of X.

Answer: Solved in this module.

Q.18. (GTU NM 2020, 4 marks): Out of 800 families with 4 children each how many families would be expected to have (i) at least one boy (ii) 2 boys and 2 girls Assume equal probabilities for boys and girls.

Answer: Solved in this module.

Q.19. (UTU 2017, 5 marks): Six dice are thrown 729 times how many do you expect at least three dice to show a five or six.

Answer: Solved in this module.

Q.20. (BPUT 2020, 6 marks): A die is tossed thrice. A success is getting 1 or 6 on a toss, find the mean and variance of the number of successes.

Answer: Solved in this module.

Q.21. (JNTUH 2019, 5 marks): In a normal distribution, 7% of the items are under 35 and 89% are under 63. Determine the mean and variance of the distribution.

Answer: Solved in this module.

Q.22. (UTU 2016, 10 marks): If x and y are two independent normal variates with mean m_1 and m_2, variance σ_1^2 and σ_2^2 respectively then prove that x + y is also normally distributed with mean $m_1 + m_2$ and variance $\sigma_1^2 + \sigma_2^2$.

Q.23. (JNTUH 2018, 2 marks): Find the mean and variance of the probability distribution having pdf $f(x) = e^{-x}$, $x > 0$.



Q.24. (BPUT 2020, 2 marks): A book has four misprints per page (on average), what is the probability that a page open at random will have no misprints on it.

Q.25. (JNTUH 2018, 3 marks): A fair coin is tossed until a head or consecutive five tails occurs. Find the discrete probability distribution.

Answer: Solved in this module.

Q.26. (JNTUH 2019, 2 marks): Let X denotes the number of heads in a single toss of 4 fair coins. Determine P $(1 < X \leq 3)$.

Answer: Solved in this module.

Q.27. (BPUT 2020, 6 marks): 2% of the fuses manufactured by a firm are found to be defective, find the probability that a box containing 200 fuses contains (i) no defective (ii) three or more defective.

Answer: 0.982, 0.762

Q.28. (JNTUH 2018, 5 marks): A die is cast until 6 appears. What is the probability that it must be cast more than 5 times?

Hint: The same as the probability of "no six" five times. "6" is one of six possible outcomes, so the probability of "no six" is 5/6. For this to happen five times the probability is $(5/6)^5$

Q.29. (JNTUH 2006, 2007, 2011, 2012, 2018, 5 marks): The marks obtained in mathematics by 1000 students is normally distributed with mean 78% and standard deviation 11%.

(i) What was the highest mark obtained by the lowest 25% students?

(ii) Within what limit did the middle 90% of the student lie?

Answer: Similar problem is solved in this module.

Q.30. (JNTUH 2018, 5 marks): The mean voltage of a battery is 15 and standard deviation 0.2. Find the probability that four such batteries connected in series will have a combined voltage of 60.8 or more volts.

Answer: Solved in this module.

Q.31. (JNTUH 2019, 5 marks): A sample of 4 items is selected at random from a box containing 12 items of which 5 are defective. Find the expected number of defective items.

Answer: Solved in this module.

Q.32. (UTU 2017, 5 marks): The mean height of 500 students is 151 cm and the standard deviation is 15 cm. Assuming that the heights are normally distributed, find how many students have heights between 120 cm and 155 cm.

Answer: Solved in this module.

Q.33. (UTU 2013): Five defective bulbs are accidently mixed with twenty good ones. It is not possible to just look at a bulb and tell whether or not it is defective. Find the probability distribution of the number of defective bulbs, if four bulbs are drawn at random from this lot.

Answer: Solved in this module.

Q.34. (JNTUH 2003, 2018, 5 marks): The diameter of motor shafts in a lot has a mean of 0.249 inch and a S.D of 0.003 inch. The inner diameter of bearings in another lot have a mean of 0.255 inch and a S.D of 0.002 inch.

(i) What arc the mean and the S.D of the clearances between shafts and bearings selected from those lots ?



(ii) If a shaft and a bearing are selected at random, what is the probability that the shaft will not fit inside, the bearing ? Assume that both dimensions arc normally distributed.

Answer: Solved in this module.

